MVM CLASSES, SCO 145, SECTOR- 24-D, CHANDIGARH. 09815002410

QUESTION PAPER: Applied maths for 2nd sem.

PART-A

Q1 Find general solution to the differential equations:

(i) $(1+y^2)dx+(x-\tan^{-1}y)dy=0$

(ii) $[(x+1)^4+2\sin y^2]dx-2y(x+1)\cos y^2dy=0$

(iii) $(x\sqrt{x^2+y^2}-y)dx+(y\sqrt{x^2+y^2-x})dy=0$

(iv) $(y^2e^{xy^2}+4x^3)dx+(2xye^{xy^2}-3y^2)dy=0$

2(a) Find the general solution of the following non- homogeneous differential equation using method of Variation of Parameters: $(D^2+6D+9)y=16\frac{e^{-3x}}{x^2+1}$.

(b) Find a power series solution of the differential equation: $y''-xy'+y=0$.

3(a) Prove that $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}dt=\frac{1}{4}\ln 5$.

(b) Evaluate $\int_0^\infty te^{-t}\sin^4 tdt=\sqrt{\pi}$.

(c) Find the inverse Laplace transform of the following functions:

(i) $\frac{9}{s^2}\left(\frac{s+1}{s^2+9}\right)$ (ii) $\frac{3(1-e^{-\pi s})}{s^2+9}$ (iii) $\ln\left(1+\frac{w^2}{s^2}\right)$.

4(a) Solve $ty''+2y'+ty=\cos t, y(0)=1$ by Laplace transformations.

(b) State and prove the convolution theorem for Laplace transforms. Using it, find the inverse of the function $F(s)=\frac{s}{(s^2+4)^2}$.

PART-B

5(a) Find the Fourier series of the following function defined by

$f(x)=\begin{cases}-k \text{ if } -\pi<x<0\\ k \text{ if } 0<x<\pi\end{cases}$ with $f(x+2\pi)=f(x)\forall x\in R.$

Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}.........=\frac{\pi}{4}$.

(b) Find the Fourier Cosine and Sine integrals of $f(x)e^{-kx}(x>0,k>0)$.

6(a) Find the complex Fourier series of $f(x)=e^x,-\pi<x<\pi$ and $f(x+2\pi)=f(x)$. Obtain from it the usual Fourier series.

(b) Find the Fourier transform of $e^{-ax^2}, a>0$. It may be assumed that $\int_{-\infty}^{\infty}e^{-v^2}dv=\sqrt{\pi}$.

7(a) Find the partial differential equations whose solutions are given by the following curves:

(i) $ax^2+by^2+z^2=1$, a, b are arbitrary constants.

(ii) $f(x^2+y^2+z^2, z^2-2xy)=0$, f is an arbitrary function.

(b) Solve the partial differential equations:

(i) $(bz-cy)p+(cx-az)q=ay-bx$

(ii) $xp-yp=x^2-y^2$.

8(a) Find the D' Alembert's solution of the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ satisfying the initial conditions $y(x,0) = f(x), \frac{\partial y}{\partial g}(x,0) = g(x)$. Here *y=y(x,t).*

(b) Solve the heat equation $\frac{\partial u}{\partial t} = \beta \frac{\partial^2 u}{\partial x^2}, 0 < x < L, t > 0$ subject to the conditions $u(0,t) = u(L,t) = 0, t > 0$ and $u(x,0) = f(x), 0 < x < L$. Here *u= u(x,t).*