MVM CLASSES, SCO 145, SECTOR- 24-D, CHANDIGARH. 09815002410

QUESTION PAPER: Applied maths for 2nd sem

PART- A

Q:1(a) Solve the differential equation $(D^4 + D^2n^2 + n^4)y = \cos mx$, where *m* and *n* are integers.

(b)Find the principle integral of the differential equation: $(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6(x^2+1)^2$ given that $y(x) = c_1x + c_2(x^2-1)$ is the general solution of the corresponding homogeneous equation.

2(a) given that $y = e^{2x}$ is a solution of $(2x+1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0$, find the complete solution by reducing the order of the given differential equation.

(b) Find a power series solution of $y'' + x^2y = 2 + x + x^2$ about *x=0.*(write at least first ten terms of the series)

3(a) State the existence theorem for Laplace transforms. Show that the Laplace transform of the function $f(t) = e^{t^2}, t \geq 0$ does not exist.

(b) Find the inverse Laplace transform of the following functions:

(i) $\sum_{k=1}^{4} \frac{(k+1)^2}{s+k^2}$ (ii) $s^{-2} - (s^{-2} - s^{-1})e^{-s}$ (iii) $\frac{1}{(s^2+w^2)^2}$.

4(a) Use Laplace transforms to solve the initial value problem $y'' + 9y = r(t)$ where $r(t) = 8\sin t$ if $0 < t < \pi$ and 0 if $t > \pi$; $y(0) = 0, y'(0) = 4$.

(b) State and prove the convolution theorem for Laplace transforms.

PART- B

5(a) Find the Fourier series of the following function defined by

$$f(x) = \begin{cases} -k \ if \ -\pi < x < 0 \\ k \ if \ 0 < x < \pi \end{cases} \text{ with } f(x+2\pi) = f(x) \forall x \in R.$$

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} .... = \frac{\pi}{4}$.

(b) Find the Fourier transform of $e^{-ax^2}, a > 0$. It may be assumed that $\int_{-\infty}^{\infty} e^{-v^2} dv = \sqrt{\pi}$.

6(a) Find the Fourier Cosine and Sine integrals of $f(x)e^{-kx} (x > 0, k > 0)$.

(b) Find the general solution of the differential equation $x(z+2a)p + (xz + 2yz + 2ay)q = z(z+a)$. Find also the integral surface which passes through the curve $y = 0, z^3 + x(z+a)^2 = 0$.

7(a) Form a partial differential equation by eliminating the arbitrary function *f* from the equations

(i) $z = f\left(\frac{xy}{z}\right)$ (ii) $z = y^2 + 2f\left(\frac{1}{x} + \ln y\right)$.

(b) Find the general solution of the equation $2x(y+z^2)p + y(2y+z^2)q = z^3$ and deduce that $yz(z^2 + yz - 2y) = x^2$ is a solution.

8(a) Find the solution *u(x, y)* of the equation $u_x - u_y = 0$ by the method of separation of variables.

(b) Find the D' Alembert's solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ satisfying the initial conditions $u(x,0) = f(x), u_t(x,0) = g(x)$. Here *u=u(x,t).*